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NASA Contractor Report 168198



APPLICATION OF ADVANCED RELIABILITY METHODS TO LOCAL STRAIN FATIGUE ANALYSIS

(NASA-CR-168198) APPLICATION OF ADVANCED
RELIABILITY METHODS TO LOCAL STRAIN FATIGUE
ANALYSIS Final Report (Arizona Univ.,
Tucson.) 52 p HC A04/MF A01 CSCI 20K

N83-29734

Unclas
G3/39 28314

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July 1983

Prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Lewis Research Center
Under Grant NAG3-41

NOTATION

The following symbols are used in this paper:

- A = cyclic strength coefficient;
- \bar{A} = expanding point of the Taylor's series;
- b = fatigue strength exponent;
- c = fatigue ductility exponent;
- E = modulus of elasticity;
- f = probability density function;
- $g(\mathbf{U})$ = failure function of \mathbf{U} ;
- $h(\cdot)$ = function defined by Eq. 21;
- k_t = stress concentration factor;
- k = number of normal equations;
- m = number of observations or data sets;
- N, N_0 = cycles to failure, service life in cycles;
- n = number of random design variables;
- $p(\cdot)$ = probability of;
- p_f = probability of failure;
- S = nominal stress;
- s = cyclic strain hardening exponent;
- \mathbf{U} = design parameter vector;
- \mathbf{U}_1^* = defined by Eq. 29;
- u_1 = reduced variable;
- μ' = desing point values;
- $\alpha, \hat{\alpha}$ = coefficient in polynomial equation, least square estimator;
- β = safety index;

ϵ, ϵ_a = strain range, cyclic strain amplitude;

ϵ'_f = fatigue ductility coefficient;

λ = direction cosine;

σ, σ_o = stress range, mean stress;

σ'_f = fatigue strength coefficient;

$\Phi(\cdot)$ = standard normal cdf;

Ω = failure domain.

EXECUTIVE SUMMARY

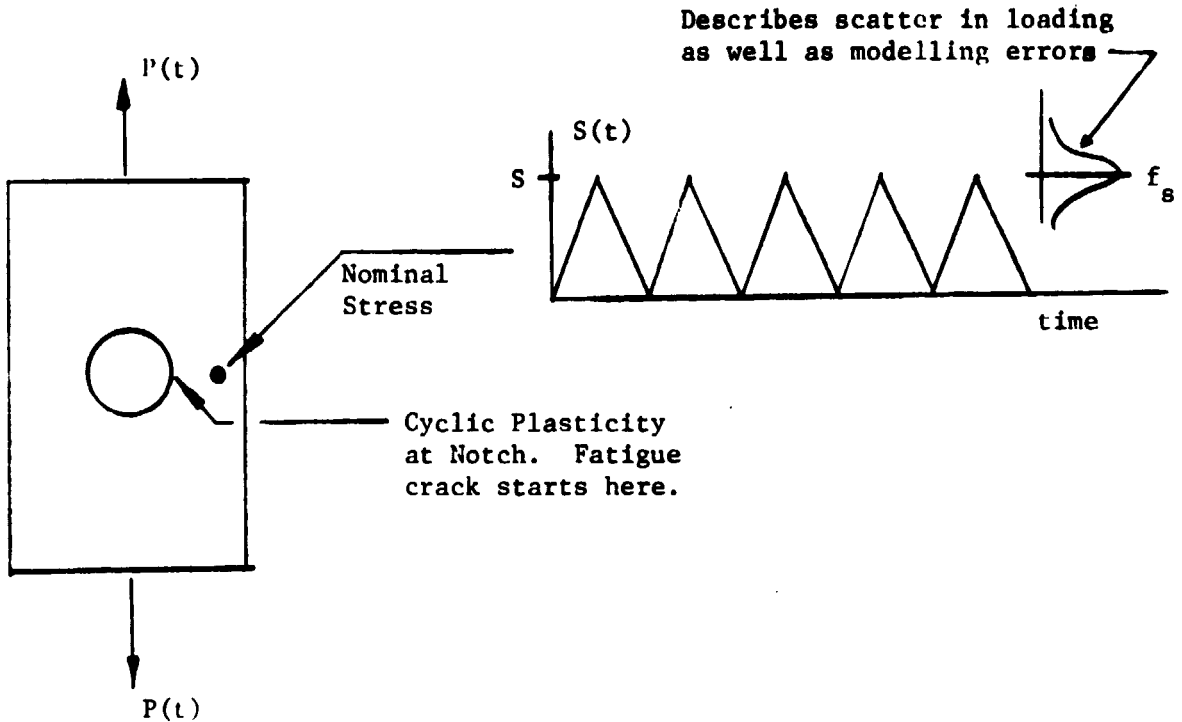
Addressed herein is the problem of computing the probability of failure for a structural component whose design factors are modelled as random variables. The relationship between the design factors for a particular mode of failure is complicated; it can be defined only using a computer algorithm, e.g., finite element analysis, local strain analysis. The situation described relates especially to fatigue risk assessment in which a local strain analysis such as strain range partitioning¹ is used to describe fatigue behavior of a component. But the procedure is general and can be applied to any mode of structural or mechanical failure.

As an example, consider the fatigue problem described in the figure. A harmonic load (zero-to-peak) is applied to the part. The stress in the bulk of the material remains elastic, but there is cyclic plasticity at the notch. Local strain analysis, requiring a computer solution, can be employed to estimate cycles to fatigue crack initiation at the notch. Random design factors include (a) the stress amplitude S reflecting statistical scatter in load data and modelling errors, and (b) the fatigue strength coefficient σ'_f and fatigue ductility coefficient ϵ'_f representing scatter in material fatigue behavior. Other sources of uncertainty could include, in general, modelling error in using Neuber's and Miner's Rule in the analysis and uncertainties in the cyclic stress strain curve.

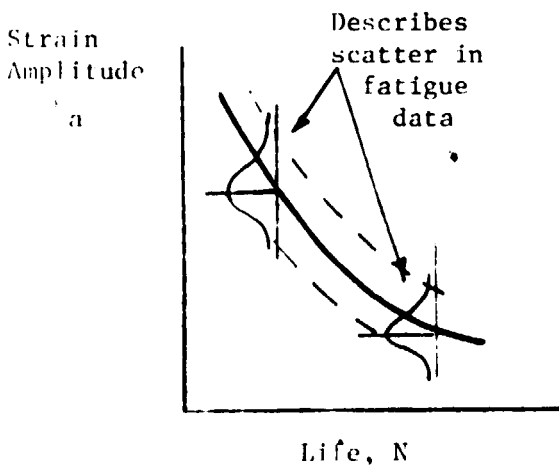
Cycles to crack initiation, N , is uncertain (a random variable) because it depends on other factors which are uncertain (random variables). The basic goal of a reliability analysis, . . . given the service life, N_0 , what is the probability of failure, i.e., the probability that N will be less than N_0 ?

A FATIGUE PROBLEM WHERE LOCAL STRAIN ANALYSIS CAN BE EMPLOYED TO PREDICT

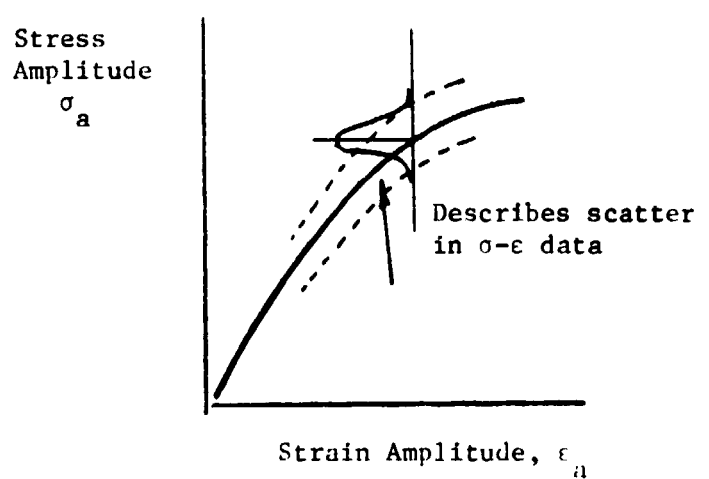
FATIGUE CRACK INITIATION



CYCLIC STRAIN-LIFE CURVE



CYCLIC STRESS-STRAIN CURVE



Computer costs for a Monte Carlo solution would be excessive because the local strain program would have to be run for each random sample point (maybe 10,000 times). An alternative proposed herein suggests a simplified scheme for establishing an approximate algebraic relationship between the design variables. Then an advanced computational method (called the Rackwitz-Fiessler algorithm) is used to estimate the probability of failure.

The procedure is much faster than Monte Carlo and probably more accurate for typical reliability problems although the latter has yet to be proven. In addition to fatigue reliability analysis, the method is thought to be particularly useful in any mechanical or structural problem where computer analysis is required to relate the random design variables.

INTRODUCTION

Numerous methods have been developed during the past decade to describe structural reliability without having to perform a multiple integration. For example, Hasofer and Lind [7] introduced the concept of generalized safety index, β , as a "measure" of reliability which used only mean and standard deviation of each design variable. However, unless the limit state is linear and all the random variables involved are normally distributed, no direct relationship between reliability and the safety index can be described in the Hasofer-Lind formulation. Distributional information is not introduced.

To extend the concept of the safety index, Rackwitz and Fiessler [12, 13] suggest that non-normal distributed variables can be transformed to equivalent normal variables in the Hasofer-Lind analysis. The refined safety index, β , and the design points are obtained through a convergent numerical process. An approximation to the actual probability of failure is provided by letting $p_f = \Phi(-\beta)$, where Φ is the standard normal distribution function. A condition is that the failure surface is approximately linear in the neighborhood of the design point. The theory behind this well-known R-F algorithm was later formally investigated by Ditlevsen [3] who called it the "principle of normal tail approximation." Chen and Lind [1] have recently refined the Rackwitz-Fiessler method by employing an additional scale parameter for the equivalent normal distribution. A more accurate approximation of the original distribution is promised.

The performance of the R-F algorithm and the C-L algorithm, both methods of "fast probability integration," have been investigated [16].

Both are considered to be effective and accurate for general reliability purposes in design. The method is fast in that numerical integration is avoided; convergence to the design point is rapid. However, the scheme becomes very inefficient or very difficult to apply in some cases where the limit state cannot be expressed in a closed form equation. An alternative is to use Monte Carlo simulation which is able to provide approximate solutions to any difficult problem. Unfortunately, Monte Carlo is costly, particularly when additional computation must be made to define the limit state. To compound the problem, very large sample sizes are required in order to get sufficient resolution in the tail areas to approximate the low probabilities typical of structural application (e.g., 10^{-3} to 10^{-5}).

To overcome the difficulties associated with Monte Carlo and a full distributional approach, a numerical method is proposed whereby the limit state is approximated as a polynomial in the neighborhood of the design point. Then the fast probability integration technique can be applied effectively to estimate P_f .

DEFINITIONS AND PROBLEM FORMULATION

Let $\underline{U} = (U_1, U_2, \dots, U_n)$ denote the vector of design factors which are, in general, random variables. It will be assumed that all U_i are uncorrelated with known distributions. The failure function, $Z = g(\underline{U})$ of random design factors is formulated so that the event of failure is,

$$\left[g(\underline{U}) \leq 0 \right] \quad (1)$$

The limit state is defined as,

$$g(U) = 0 \quad (2)$$

The probability of failure is defined as,

$$P_f = P [g(U) \leq 0] \quad (3)$$

P_f can be evaluated from the following multiple integral

$$P_f = \int_{\Omega} f_U(U) dU \quad (4)$$

where $f_U(U)$ is the joint probability density function of the random vector, U ; Ω is the failure domain in design parameter space.

In general, evaluation of Eq. 4 requires numerical integration.

The operation is particularly difficult if the number of the random variables exceeds two. And in practice, the failure function does not necessarily have to be expressed in explicit form. An example presented herein is a computation of reliability for a fatigue problem for which the local strain approach is used to determine cycles to failure in a notched member to which a constant amplitude oscillatory load is applied. Fatigue crack initiation life N at the notch (point of stress concentration) which experiences cyclic plasticity is predicted using the following three equations [2, 6]

1. Neuber's rule:

$$\sigma \epsilon = \frac{(k_t S)^2}{E} \quad (5)$$

2. Cyclic stress-strain curve:

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{A} \right)^{1/s} \quad (6)$$

3. Strain-life curve which defines fatigue strength,

$$\epsilon_a = \frac{\sigma'_f - \sigma_o}{E} (2N)^b + \left(\frac{\sigma'_f - \sigma_o}{\sigma'_f} \right)^{c/b} \epsilon'_f (2N)^c \quad (7)$$

where

- k_t = stress concentration factor
- S = nominal stress
- E = modulus of elasticity
- σ = local stress
- ϵ = local strain
- A = cyclic strength coefficient
- s = cyclic strain hardening exponent
- ϵ_a = cyclic strain amplitude
- σ_o = mean stress
- σ'_f = fatigue strength coefficient
- ϵ'_f = fatigue ductility coefficient
- b = fatigue strength exponent
- c = fatigue ductility exponent

The details of computing fatigue initiation life is described by Collins [2] and by Fuchs and Stephens [6]. The algorithm is complicated, and a computer program is required. Therefore, an algebraic expression of the failure function $g(U)$ is not immediately available.

To simplify the reliability problem, assume that only σ'_f , ϵ'_f and S (applied nominal stress range of a constant amplitude process) are random variables. The remaining parameters are constant. It will be assumed

also that the mean stress effect is negligible due to stress relaxation;
 $\sigma_0 = 0$. It is important to note that in the example which follows, it is possible, with no extension of the theory, to include, for example, k_t , A , s , and even σ_0 as random variables.

A local strain analysis computer program is the only practical way of computing cycles to failure N as a function of σ'_f , ϵ'_f , and S . A closed form expression of

$$N = H(S, \sigma'_f, \epsilon'_f) \quad (8)$$

does not exist. Given that the design life is N_0 , the event of failure is $(N \leq N_0)$ and the limit state is $(N = N_0)$.

Eqs. 5 to 7 could be expressed implicitly as:

$$S = h(\sigma'_f, \epsilon'_f, N), \text{ etc.} \quad (9)$$

If the closed form expression of h were given the R-F algorithm could be used effectively to estimate p_f given $(N = N_0)$. The scheme presented here is to construct an approximation of h in the neighborhood of the design point and then use the R-F method. Monte Carlo is used as a check on the R-F computations.

MONTE CARLO METHOD

To estimate probabilities associated with a complicated function of random variables, a Monte Carlo solution can be used. Consider

$$Z = g(\underline{U}) \quad (10)$$

where \underline{U} is a vector of random variables. The computer generates a random sample of \underline{U}_i ; $i = 1, J$, where J is typically a large number, e.g., 10,000.

Eq. 10 is employed to obtain a random sample of Z_1 of size J . To estimate $P(Z \leq 0)$, for example, one can take the number of Z_1 having a value less than zero and divide by J . Moreover, the empirical cumulative distribution function (CDF) of Z can be plotted on probability paper.

It is a well known fact that the Monte Carlo solutions tend to be expensive relative to the accuracy obtained. Often a large sample size is required, particularly in structural reliability analysis when the values of the probability of failure are typically small. For example, it might require 10^4 samples to provide a reasonable estimate of probability of failure in the range of $p_f > 10^{-3}$. If the failure function is complicated so that a costly analysis is required for each Z_1 , Monte Carlo becomes impractical. Nevertheless, the method is easy to apply, able to solve complicated problems, and therefore is a valuable tool for design and for research.

A limited Monte Carlo analysis is employed later to check the results of the proposed method.

FAST PROBABILITY INTEGRATION TECHNIQUE

The Rackwitz-Fiessler (R-F) algorithm, used herein, is well documented in literature [9, 12, 13, 14]. The following procedures are the summary of the method.

1. Transformation of the basic variables to the reduced variables,

$$u_1 = \frac{U_1 - \mu_1}{\sigma_1} \quad (11)$$

where μ_1 and σ_1 are the mean and the standard deviation of U_1 respectively. Note that u_1 are random variables having a mean of zero and standard deviation of unity.

2. Determine the minimum distance, β , from the origin to the limit state on the reduced coordinates.
3. Obtain the "equivalent normal" values of μ_1 and σ_1 for those random design factors which do not have a normal or lognormal distribution. This is done by imposing the conditions that the density and distribution functions of the equivalent normal are the same as the basic variable at the design point.
4. Repeat steps 1. to 3. until β converges.

The point on the limit state closest to the origin is called the design point. β is called the safety index and is considered to be a measure of reliability because it is inversely proportional to the probability of failure.

It is a common practice to calculate β by iteratively solving the following equations [9, 14]

$$\lambda_1 = - \left(\frac{\partial g}{\partial u_1} \right)_{\mu'} / \sqrt{\sum_{j=1}^n \left(\frac{\partial g}{\partial u_j} \right)_{\mu'}^2} \quad (12)$$

$$u_1' = \beta \lambda_1 \quad (13)$$

$$g(\lambda_1 \beta, \lambda_2 \beta, \dots, \lambda_n \beta) = 0 \quad (14)$$

where the partial derivatives are calculated at the design point, μ' ; λ_1 are the direction cosines of the vector that define the design point.

Note that this scheme requires computation of partial derivatives of the failure function, g . In the case when the failure function cannot be expressed explicitly, the method becomes inefficient. Furthermore, in many practical situations the failure function may not be explicitly differentiable, and it is necessary to use numerical differentiation. Again the failure function needs to be known.

β can also be computed by solving the constrained optimization problem:

$$\beta = \min. \sqrt{\sum_{i=1}^n u_i^2} \quad (15)$$

$$g(\mu) = 0 \quad (16)$$

The computations of partial derivatives are avoided (e.g., see Ref. 11). For a complicated failure function, it is suggested that this scheme be adopted and the failure function be approximated by a closed form algebraic expression so that β and μ' can be effectively obtained.

A probability of failure computed by

$$p_f = \Phi(-\beta) \quad (17)$$

provides a reasonable estimate of the "exact" value of p_f for many cases [4, 16]. A study of the R-F algorithm has demonstrated that for many cases, (a) estimate of p_f are "good," to "excellent" and (b) R-F computer costs are a fraction of Monte Carlo, e.g., 1/50 for some examples [16].

POLYNOMIAL APPROXIMATION TO THE FAILURE FUNCTION

If the failure function can be approximated by a simple (e.g., polynomial) equation, β can be determined effectively by solving the optimization problem (Eqs 15 and 16). The entire process of computing β and then

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p_f is summarized by the block diagram of Fig. 1. The method will be developed and demonstrated by solving a sample problem.

Assume, in general, that there are $n + 1$ variables involved in a failure function expression

$$g(U_0, U_1, \dots, U_n) = 0 \quad (18)$$

Assume also that the failure function is differentiable with continuous partial derivatives in the whole domain. By expanding it at a point $a = (a_0, a_1, \dots, a_n)$ by Taylor series, the following polynomial results:

$$\begin{aligned} g(U) = g(a) + \sum_{i=0}^n \frac{\partial g}{\partial U_i} \bigg|_a (U_i - a_i) + \frac{1}{2} \sum_{i=0}^n \frac{\partial^2 g}{\partial U_i^2} \bigg|_a (U_i - a_i)^2 \\ + 2 \sum_{i=0}^{n-1} \sum_{j=i+1}^n \frac{\partial^2 g}{\partial U_i \partial U_j} \bigg|_a (U_i - a_i) (U_j - a_j) + \text{H.O.T. (higher order terms)} \end{aligned} \quad (19)$$

In order to compute β using the optimization scheme of the R-F algorithm, it is necessary to rewrite the equation such that any one of the variables is a function of the remaining variables. Thus Eq. 18 becomes, when one chooses U_0 ,

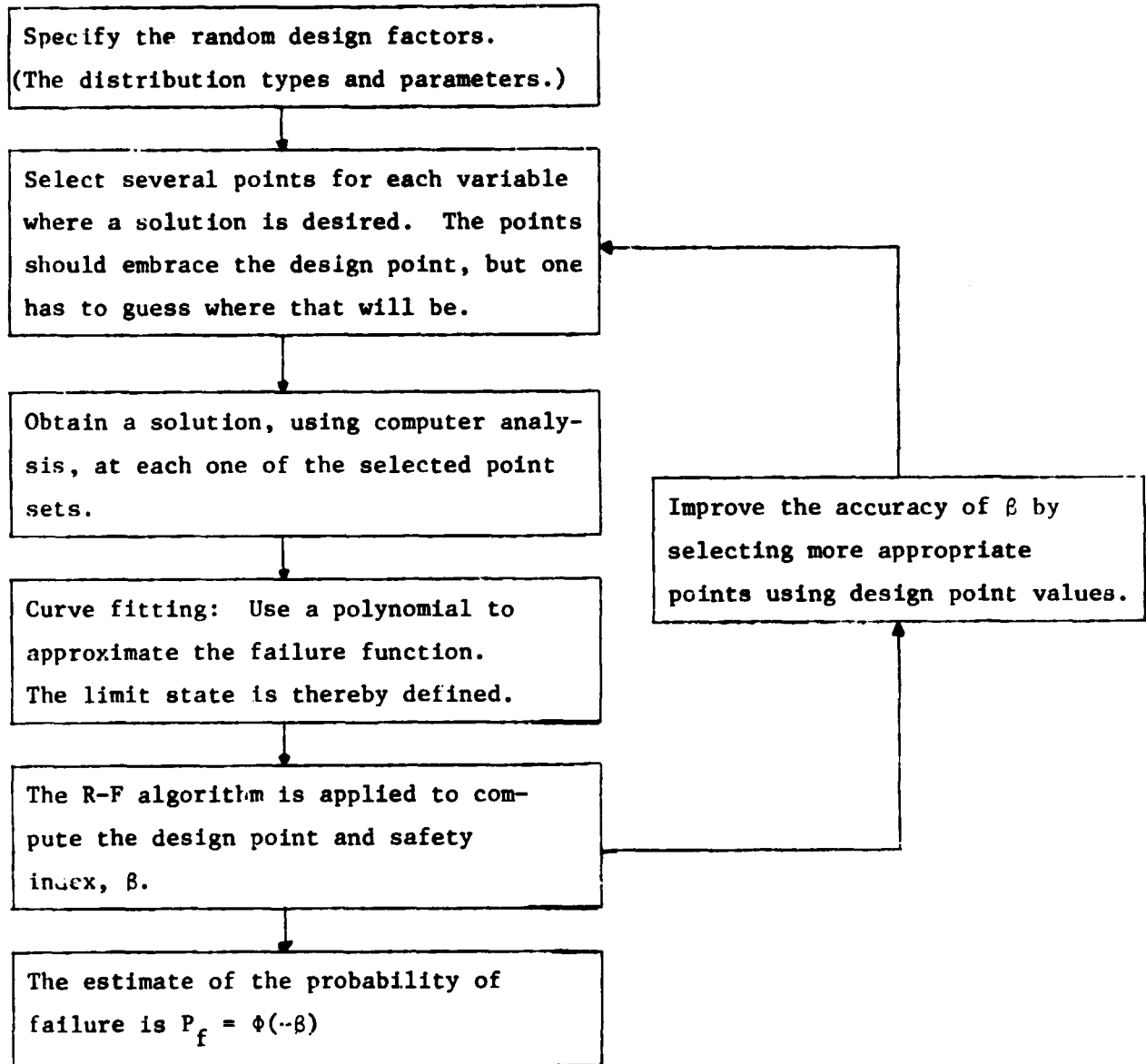
$$U_0 = h(U_1, U_2, \dots, U_n) \quad (20)$$

such that the failure function is

$$g(U) = U_0 - h(U_1, U_2, \dots, U_n) = 0 \quad (21)$$

By substituting Eq. 21 into Eq. 19, it follows that

FIGURE 1. Block Diagram for Computation of the Safety Index, β , when the Failure Function is Not Available as a Closed Algebraic Form.



$$U_0 = h(a) + \sum_{i=1}^n \frac{\partial h}{\partial U_i} \bigg|_a (U_i - a_i) + \frac{1}{2} \sum_{i=1}^n \frac{\partial^2 h}{\partial U_i^2} \bigg|_a (U_i - a_i)^2$$

$$+ 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{\partial^2 h}{\partial U_i \partial U_j} \bigg|_a (U_i - a_i)(U_j - a_j) + \text{H.O.T.} \quad (22)$$

Simplifying,

$$U_0 = \alpha_0 + \sum_{i=1}^n \alpha_i (U_i - a_i) + \sum_{i=1}^n \alpha_{n+i} (U_i - a_i)^2$$

$$+ \sum_{i=1}^{n-1} \sum_{j=i+1}^n \alpha_{(n-\frac{1}{2})(i+1)+j} (U_i - a_i)(U_j - a_j) + \text{H.O.T.} \quad (23)$$

where the α 's are defined by comparing Eq. 22 and Eq. 23, Eq. 23 can be further simplified as

$$U_0 = \alpha_0 + \sum_{i=1}^k \alpha_i x_i + \text{Error (H.O.T.)} \quad (24)$$

where

$$x_i = U_i - a_i \quad \text{for } i = 1, n$$

$$= (U_i - a_i)^2 \quad \text{for } i = n+1, 2n$$

$$= (U_i - a_i)(U_j - a_j) \quad \text{for } i = 2n+1, k = [(n+2)(n+1)/2] - 1$$

This is the familiar multiple linear regression model where x_i are independent random variables. (Note: The model is linear in the parameters (the α 's) regardless of the shape of the surface it generates.) The least

square normal equation may be written in matrix notation as (see e.g. Ref. 10)

$$[X]^T [X] [\alpha] = [X]^T [U_o] \quad (25)$$

where

$$[X] = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdot & \cdot & \cdot & x_{1k} \\ \cdot & x_{21} & x_{22} & \cdot & \cdot & \cdot & x_{2k} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ 1 & x_{m1} & x_{m2} & \cdot & \cdot & \cdot & x_{mk} \end{bmatrix}, \quad (26)$$

$$[\alpha] = \begin{bmatrix} \alpha_o \\ \alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_k \end{bmatrix}, \quad [U_o] = \begin{bmatrix} (U_o)_1 \\ (U_o)_2 \\ \cdot \\ \cdot \\ \cdot \\ (U_o)_m \end{bmatrix}$$

Note that there are $k + 1$ normal equations, one for each of the unknown α_i . m is the number of observations or data sets. Given the data sets x_{ij} and $(U_o)_i$, the solution will be the least square estimators, $\hat{\alpha}_i$.

SELECTION OF THE DATA POINTS

Assume that the failure function is to be approximated by a truncated polynomial equation. In a general data analysis problem, data obtained from experiments are subject to random errors. A statistical analysis of estimators $\hat{\alpha}_i$ are required to validate the model. In such cases, accuracy of the model will be heavily dependent upon the sample size, m .

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In this study, however, the data sets $x_{ij}, (U_o)_i$ are those values which satisfy the exact failure function. Therefore, smooth functions are anticipated and statistical analysis is not necessary. The number of data sets for curve fitting can be chosen relatively small. More precisely, the minimum number of data sets (m) is equal to the number of α_i .

The key to constructing an effective approximating model is to choose, for curve fitting, data points (centered around the expanding point a) from a domain that is relatively small to give satisfactory agreement between the exact and the approximating limit state, but wide enough to cover the unknown design point. When R-F algorithm is used, the model need be accurate only in the region of the design point rather than the whole domain. If the expanding point, a , which must be chosen at the outset, is relatively close to the design point, "accurate" results can be expected.

In the general case, the design point may be difficult to predict in advance. However, it is reasonable to assume that design point values of stress variables would be greater (may be much greater) than its mean or median. Similarly, design point values of strength variables would be less than its mean. Hence, one can at least make a reasonable first guess of the neighborhood of the design point. Another observation which is useful is that the failure functions are expected to be relatively smooth and unlikely to have discontinuities or other significant nonlinearities in the neighborhood of the design point. Therefore, it seems reasonable

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to use a quadratic polynomial model to approximate the failure function. On the other hand, it is worth mentioning that the simple relationship between p_f and β of Eq. 17 is a good approximation when the limit state surface can be approximated by a tangent hyperplane at the design point [5]. In such cases it is evident that a quadratic model will be sufficient to approximate the limit state in the region close to the design point.

As an example of solving α_1 using minimum of data points, consider the case where there are four design variables involved. If the quadratic polynomial without interaction terms (or mixed terms, e.g., $(U_1 - a_1)(U_2 - a_2)$, . . . etc.) is chosen, the model is

$$U_o = \alpha_o + \alpha_1(U_1 - a_1) + \alpha_2(U_2 - a_2) + \alpha_3(U_3 - a_3) + \alpha_4(U_1 - a_1)^2 + \alpha_5(U_2 - a_2)^2 + \alpha_6(U_3 - a_3)^2 \quad (27)$$

The expanding point \bar{a} has been chosen and it is required to compute the estimates, $\hat{\alpha}_o . . . \hat{\alpha}_6$. These estimates can be computed by calculations relating the design parameters at the following specific points.

$\underline{U_1}$	$\underline{U_2}$	$\underline{U_3}$	$\underline{U_o}$
a_1	a_2	a_3	$(U_o)_1$
$a_1 + \Delta U_1$	a_2	a_3	$(U_o)_2$
$a_1 - \Delta U_1$	a_2	a_3	$(U_o)_3$
a_1	$a_2 + \Delta U_2$	a_3	$(U_o)_4$
a_1	$a_2 - \Delta U_2$	a_3	$(U_o)_5$
a_1	a_2	$a_3 + \Delta U_3$	$(U_o)_6$
a_1	a_2	$a_3 - \Delta U_3$	$(U_o)_7$

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The ΔU_1 's must be chosen. They must be large enough to embrace the design point, but not too large so as to introduce significant errors in the polynomial approximation. Here the analyst must again use his intuition.

Finally, the estimates of α 's are computed by noting the following. $\hat{\alpha}_0$ is the value of U_0 at the expanding point; $\hat{\alpha}_1$ and $\hat{\alpha}_4$ can be determined when $U_2 = a_2$ and $U_3 = a_3$, . . . etc. Thus, $\hat{\alpha}_1$ can be determined from the above seven data sets by choosing appropriate values of $\pm \Delta U_1$ which define the range of data for each variable.

In general, one can solve $\hat{\alpha}_1$ using matrix operations (Eq. 25), where x matrix for the example is

$$[X] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \Delta U_1 & 0 & 0 & \Delta U_1^2 & 0 & 0 \\ 1 & -\Delta U_1 & 0 & 0 & \Delta U_1^2 & 0 & 0 \\ 1 & 0 & \Delta U_2 & 0 & 0 & \Delta U_2^2 & 0 \\ 1 & 0 & -\Delta U_2 & 0 & 0 & \Delta U_2^2 & 0 \\ 1 & 0 & 0 & \Delta U_3 & 0 & 0 & \Delta U_3^2 \\ 1 & 0 & 0 & -\Delta U_3 & 0 & 0 & \Delta U_3^2 \end{bmatrix} \quad (28)$$

If the interaction terms are included, a minimum of three more data sets are required to determine $\hat{\alpha}_7$ to $\hat{\alpha}_9$. As an example,

U_1	U_2	U_3	U_0
$a_1 + \Delta U_1$	$a_2 - \Delta U_2$	0	$(U_0)_8$
$a_1 - \Delta U_1$	0	$a_3 + \Delta U_3$	$(U_0)_9$
0	$a_2 + \Delta U_2$	$a_3 - \Delta U_3$	$(U_0)_{10}$

Similar tables can be easily constructed when the number of design variables $(n + 1)$ is greater than four. In general, the minimum number of data sets required is $2n + 1$ if the interaction terms are excluded, and is $(n + 2)(n + 1)/2$ for a complete quadratic equation.

When using the minimum number of data points to generate the model, the values at the data points of the model are exact. Then the curve fitting procedure described above is actually an effort of trying to approximate the exact surface by interpolation. To determine suitable values of the three data points for each variable, namely, the expanding point and the other two points which define the "interpolation range," consider the following example. Assume that a stress random variable U has a known distribution, but no information is available about the value of the design point. As shown in Fig. 2, a decision is made to choose the interpolation range from the median of U (data point 1) to the value of U where its CDF is 0.9999 (data point 3), and let the expanding point to be in the middle (data point 2). For a strength variable, points 2 and 3 should be chosen at the left tail region. In doing this, it is anticipated (as a first approximation) that the design point will lie within the interpolation range. To improve the approximation, a new range may be chosen if the design point lies outside this range in the first reliability computation by the fast probability integration technique. As demonstrated later, it is thought that one or two tries will be sufficient for generating satisfactory results.

In the first attempt, in general, one would like to choose a wide interpolation range to ensure that the range will cover the design point. However, it is obvious that a wider interpolation range may result in poorer approximation. In addition, if significant nonlinearities exist in the range, the quadratic curve or surface may turn into an inappropriate direction very fast which may result in a problem of obtaining

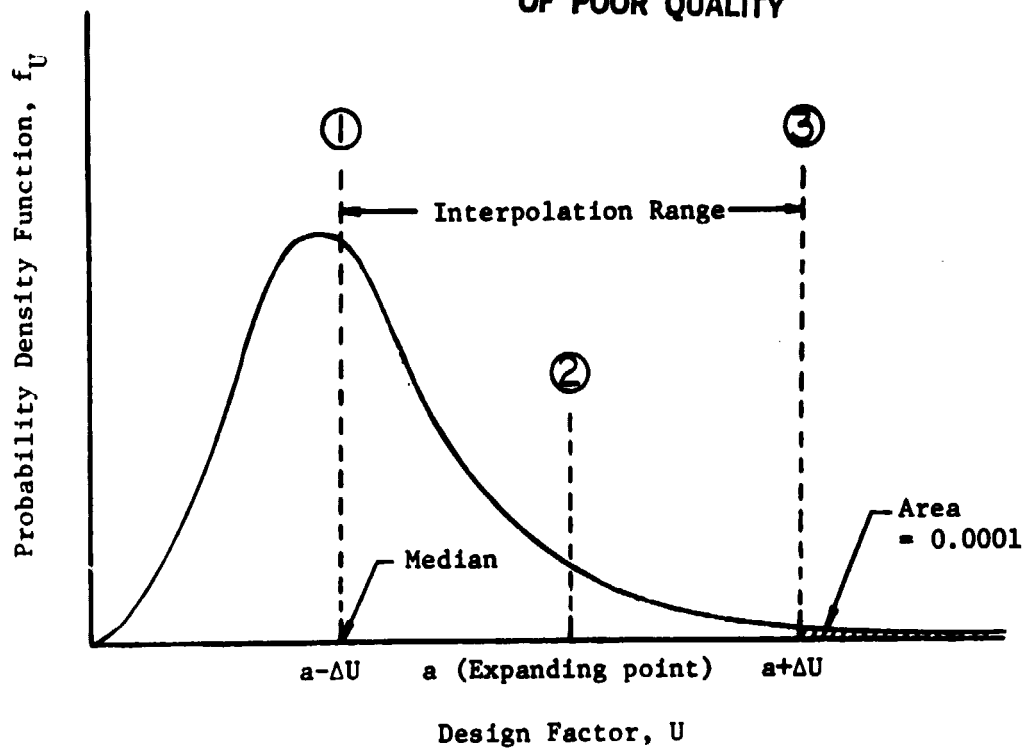


FIGURE 2. An Example of Choosing Data Points for Generating Curve Fitting Model (Stress Variable Case).

unreasonable design point. It is helpful to have a good physical feel of the problem. Consider the example of fatigue reliability analysis mentioned before. For a given life N , fatigue strength (defined by ϵ) should increase if σ_f' being increased. In other words, $\partial\epsilon/\partial\sigma_f'$ should always be positive. Suppose a linear polynomial model is chosen. As a first approximation, the data points $(\epsilon_1, \sigma_f', \dots)$ will be fitted by a straight line on the two dimensional coordinates plane (e.g., $\epsilon - \sigma_f'$ plane). But the data are exact, and it follows that the sign of $\partial\epsilon/\partial\sigma_f'$ will be preserved. Hence, the linear model is excellent in preserving the basic functional relationships between strength and stress variables.

In general, however, the quadratic model is more appropriate because of the wide range of good fit of the failure surface. But it is also clear that the slopes or (first) partial derivatives of a second degree equation can be positive, negative, or zero. Therefore, when a "strictly monotone" nonlinear function is approximated by a quadratic equation, it is no longer a strictly monotone function. As a result, there exist certain domains in which a strength variable will behave like a stress variable, and vice versa. Consequently, there is a possibility that the design point will converge to the undesirable domain where very poor approximation of the limit state is unavoidable, resulting in unreasonable design point. Therefore, it is important to examine the model before calculating β .

Instead of checking the result of the curve fitting by plotting, it is more efficient to calculate the "effective domain" of the model where the basic functional relationships mentioned above are preserved. As

illustrated in Fig. 3 the effective range can be determined for each variable by calculating the value at which the partial derivative is zero. For instance, recall Eq. 27 for a quadratic polynomial with no interaction terms. Consider U_0 as the dependent variable, and U_1, U_2, U_3 as the independent variables. From the conditions of $\partial U_0 / \partial U_1 = 0$, the "critical points" of U_0 , denoted by U_1^* , are

$$U_1^* = a_1 - \frac{\alpha_1}{2\alpha_4} \quad (29)$$

$$U_2^* = a_2 - \frac{\alpha_2}{2\alpha_5} \quad (30)$$

$$U_3^* = a_3 - \frac{\alpha_3}{2\alpha_6} \quad (31)$$

Thus, the effective range can be easily determined. Similar results can be immediately inferred when more design variables are involved.

By introducing interaction terms to form a complete quadratic equation, Eq. 29 is replaced by

$$U_1^* = \left(a_1 - \frac{\alpha_1}{2\alpha_4} \right) - \frac{\alpha_7}{2\alpha_4} (U_2 - a_2) - \frac{\alpha_8}{2\alpha_4} (U_3 - a_3) \quad (32)$$

Considering the extremes of the interpolation range where $U_1 = a_1 \pm \Delta U_1$ for U_2 and U_3 , Eq. 32 becomes,

$$U_1^* = \left(a_1 - \frac{\alpha_1}{2\alpha_4} \right) - \frac{\alpha_7}{2\alpha_4} (\pm \Delta U_2) - \frac{\alpha_8}{2\alpha_4} (\pm \Delta U_3) \quad (33)$$

therefore, at least at one of the extremes, U_1^* will be

$$U_1^* = \left(a_1 - \frac{\alpha_1}{2\alpha_4} \right) - \left| \frac{\alpha_7}{2\alpha_4} \Delta U_2 \right| - \left| \frac{\alpha_8}{2\alpha_4} \Delta U_3 \right| \quad (34)$$

which is at best, equal to the value obtained from Eq. 29. Similar arguments can be used for the other U_i^* , and for the case of more variables.

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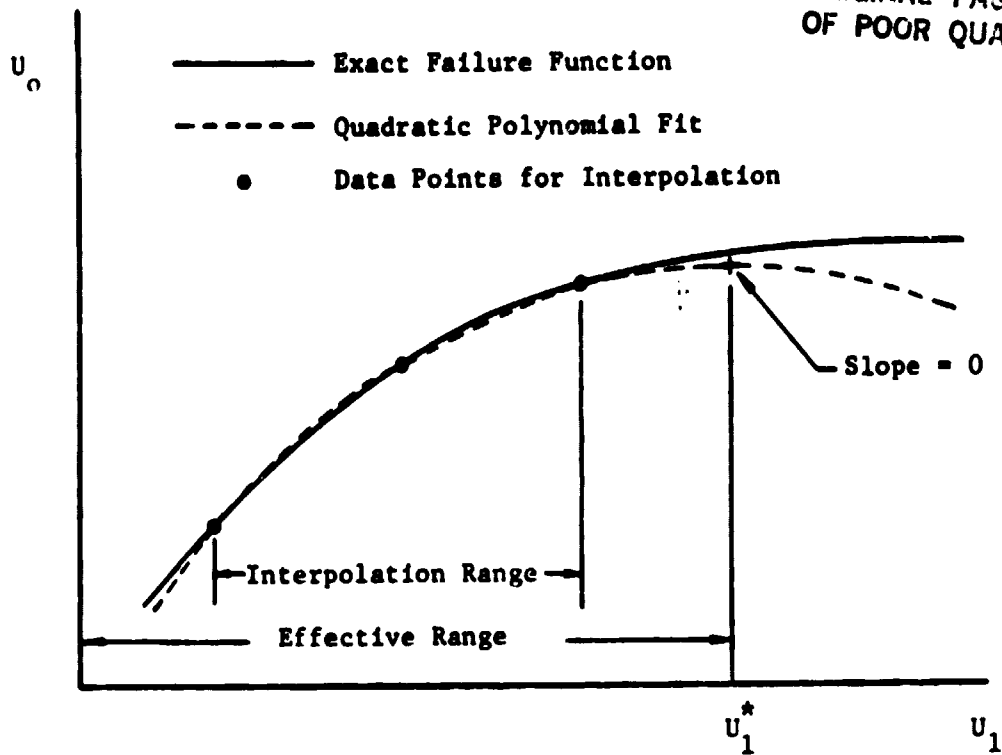


FIGURE 3. Definition of the Effective Range for the Quadratic Polynomial Curve Fitting Model.

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Thus, while the introduced interaction terms have the promises of improving the accuracy of the model, the effective range becomes smaller.

A general rule regarding the quality of the polynomial expression is that the model is good if all U_1^* are located well outside the interpolation range (See Fig. 3). It is clear that the effective range will be much wider than the interpolation range if the failure function is approximately linear in the interpolation range. On the other hand, when U_1^* lies inside the data range, it is implied that significant nonlinearity exists in the interpolation range. A "bad fit" results in some region of the interpolation range. Likely the fit is worse in the "extrapolation range."

Using the information of effective range, there are several ways of overcoming this "nonlinearity problem": (a) Restrict the value of the variable within its effective range. This will be demonstrated in the following example of the fatigue life reliability analysis; (b) Use linear model as a first approximation followed by a quadratic model using more appropriate interpolation range; (c) Use a piecewise (splines) model by splitting the interpolation range. In the limiting case, any curve can be considered as connected by a series of straight lines.

The methods above are useful when a range of p_f need be estimated, and a single quadratic curve is unable to provide a good fit in the relatively wide domain because of the nonlinearity of the failure function. For the case of single design-point checking problem, "range reduction" can be very effective simply because a narrower interpolation range always provides a better effective range.

In conclusion, it is thought that generally the quadratic polynomial curve fitting is an appropriate method to approximate the failure function.

Furthermore, because the interaction terms tend to reduce the effective range and more data point sets are required for modeling, it is recommended that a quadratic polynomial model without the interaction terms may be adopted for an efficient estimation of p_f and the design point values. Using a minimum amount of data points, this model will be applied in the following example to demonstrate its effectiveness. It will be shown that critical values, U_1^* , are useful for validating and improving the model. Finally, it is believed that accurate β can be effectively obtained through the "design-point searching" and the "interpolation-range reduction" processes regardless of the complexity and nonlinearity of the failure function.

EXAMPLE - FATIGUE LIFE RELIABILITY ANALYSIS

Demonstrated here is the general method of producing a quadratic polynomial model where interaction terms are neglected. The sample problem involves fatigue reliability using local strain analysis to predict life. A zero-to-peak constant amplitude load producing the nominal stress history as shown in Fig. 4 is applied to the notched member. The peak stress S is not known exactly, and is considered to be a random variable. An illustration of the basic equations of local strain analysis (Eqs. 5 to 7) are provided in Figs. 5 and 6. Fig. 5 shows that σ and ϵ are determined from Neuber's rule and the cyclic stress-strain curve. Fig. 6 illustrates the relationship between strain amplitude at the notch (ϵ_a) and cycles to failure (N) for a given σ'_f and ϵ'_f . Both σ'_f and ϵ'_f are considered to be random variables, describing the scatter in fatigue data.

Three different applied stress conditions are examined using the proposed method described herein. The results are compared with those using

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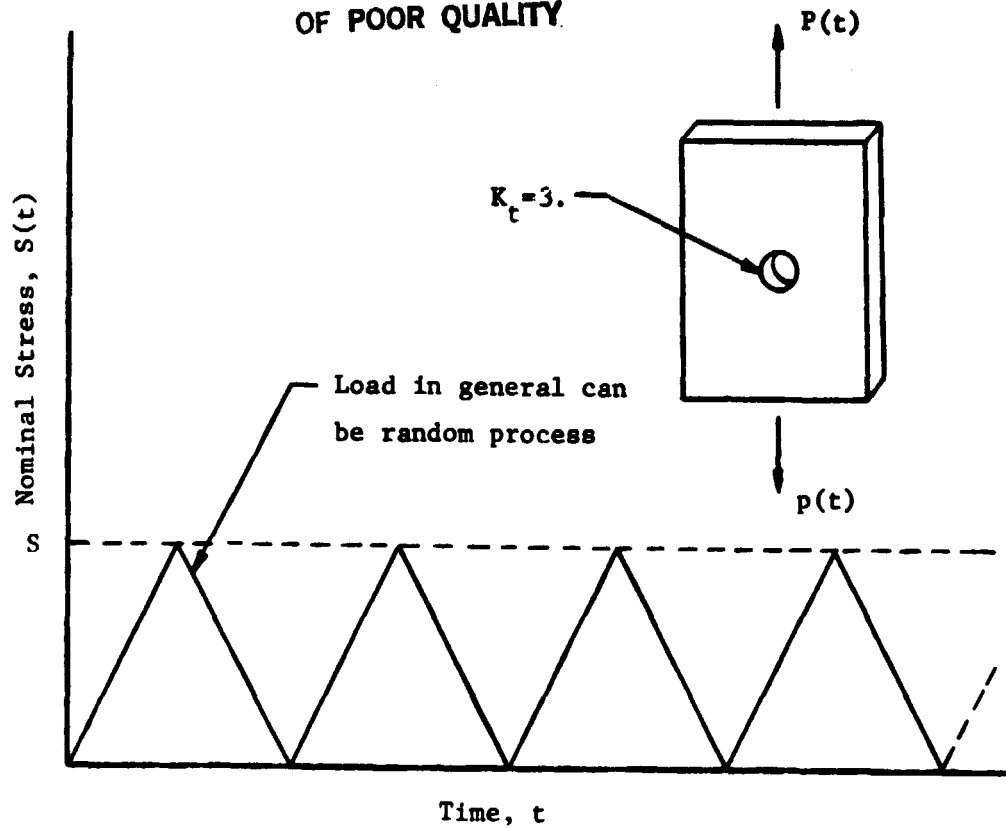


FIGURE 4. Nominal Stress Applied to Notched Member.

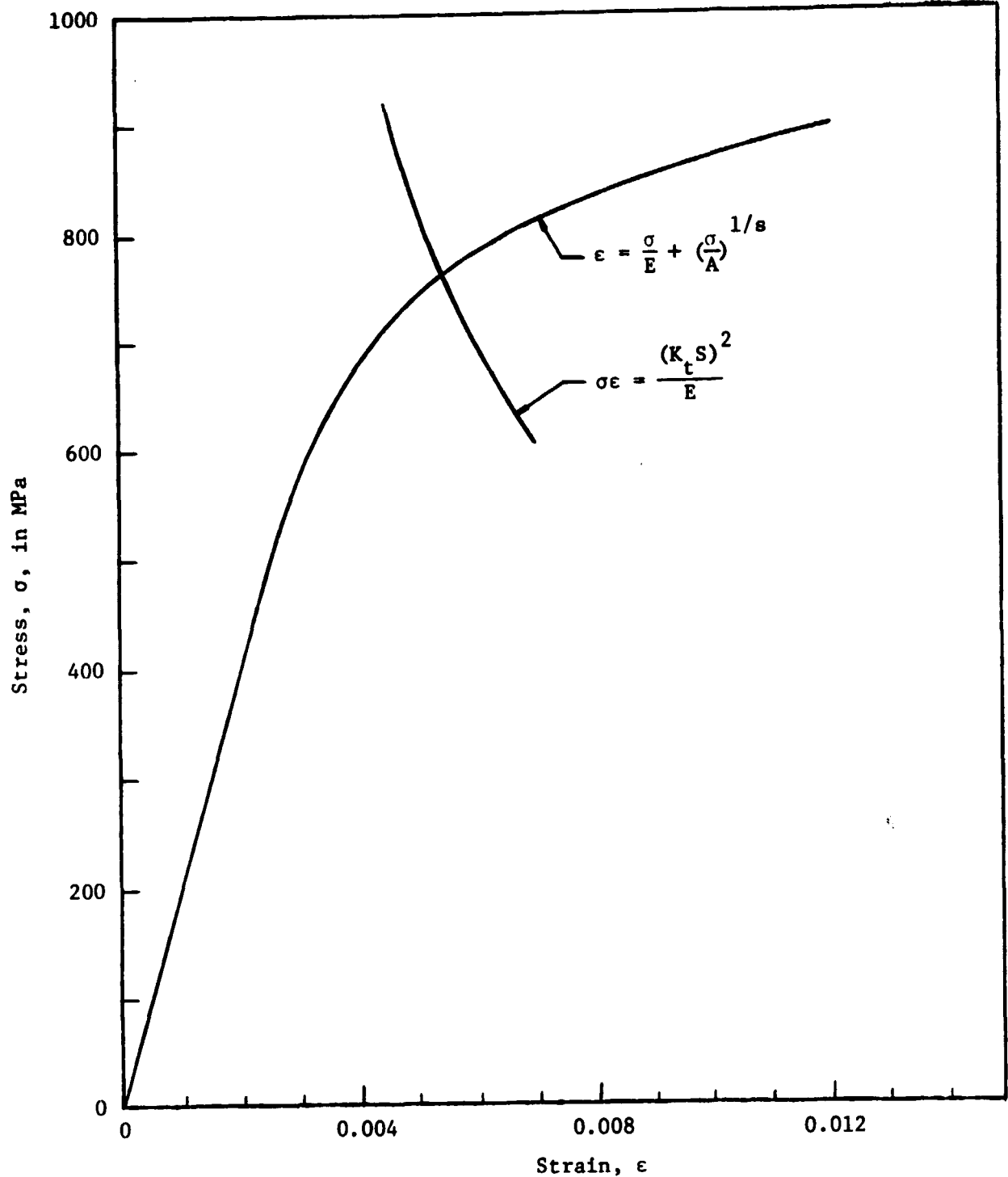


FIGURE 5. Cyclic Stress-Strain Curve and Neuber's Rule (Case A, $S = 310.2\text{MPa}$).

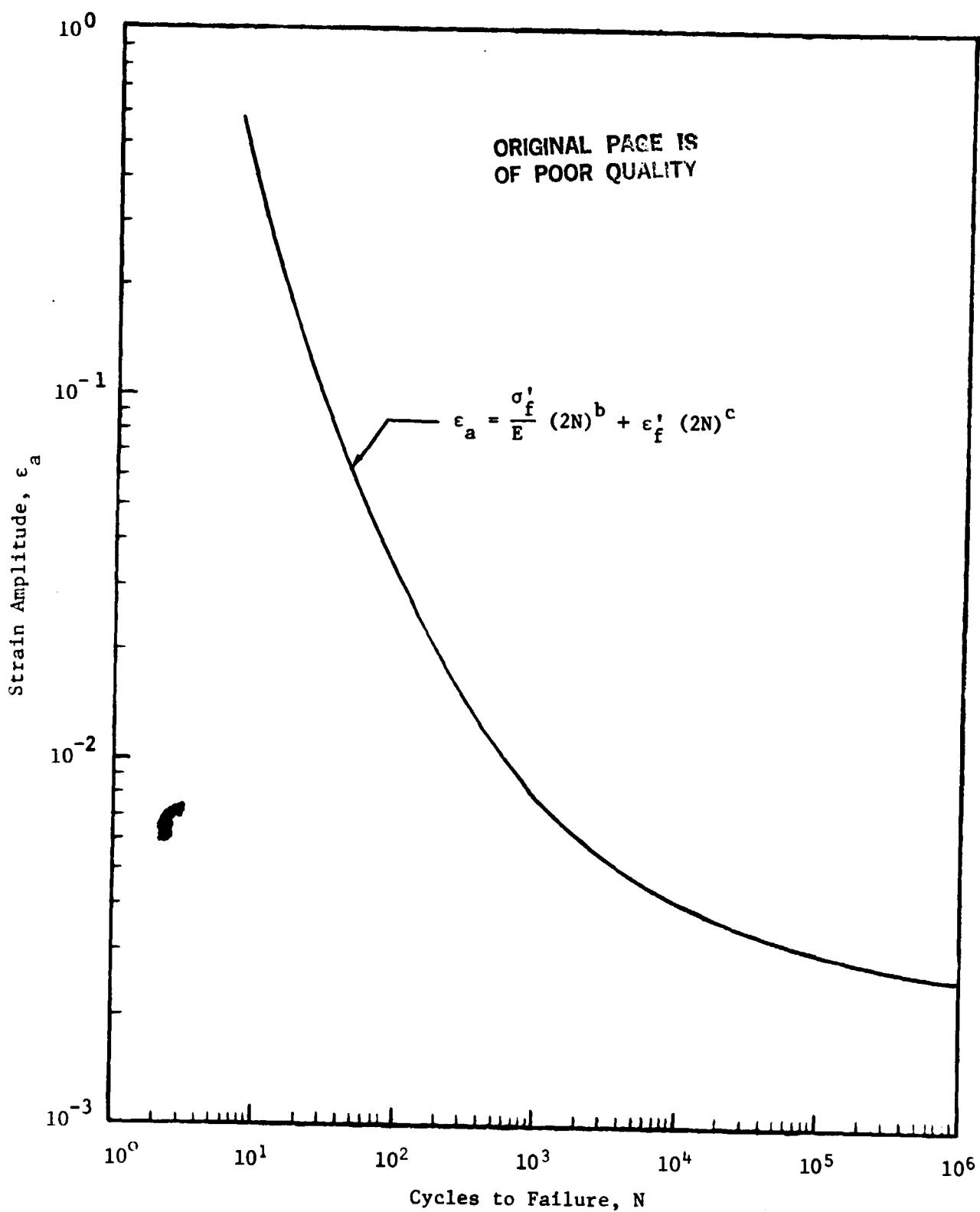


FIGURE 6. Strain Life Curve for Case A (Using Median Value of S , σ'_f and ϵ'_f).

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Monte Carlo method where 100 samples are generated in each case to plot the empirical CDF.

Case A:

The material analyzed is Waspaloy B at 1000° F. The median nominal stress is $\bar{S} = 310$ MPa (45 ksi), The data is tabulated in Table 1. To obtain wide range of effectiveness of the polynomial fit, data points are chosen as described in Fig. 7.

Cycles to failure N is computed using a local strain analysis program. Seven data sets for curve fitting (analysis is repeated seven times) are given in Table 2. The scale of N has been changed for easier data handling. Once the data sets are generated, any variable can be treated as the dependent variable. In this example, S was chosen,

$$S = \alpha_0 + \alpha_1 (\sigma'_f - a_1) + \alpha_2 (\epsilon'_f - a_2) + \alpha_3 (\log N - a_3) + \alpha_4 (\sigma'_f - a_1)^2 + \alpha_5 (\epsilon'_f - a_2)^2 + \alpha_6 (\log N - a_3)^2 \quad (35)$$

where the expanding point values, a_1 , can be found in Table 2. By the transformation of $x_1 = \sigma'_f - a_1$, etc., as defined in Eq. 24, S can also be written as

$$S = \alpha_0 + \alpha_1 x_1 + \dots + \alpha_6 x_6 \quad (36)$$

Therefore, α_1 can be solved simultaneously from Eq. 25 where the X matrix can be constructed using data in Table 2. The result is:

$$S = 497.7 + 0.268 (\sigma'_f - a_1) + 20.8 (\epsilon'_f - a_2) - 168 (\log N - a_3) + 0.000027 (\sigma'_f - a_1)^2 - 4.52 (\epsilon'_f - a_2)^2 + 37.5 (\log N - a_3)^2 \quad (37)$$

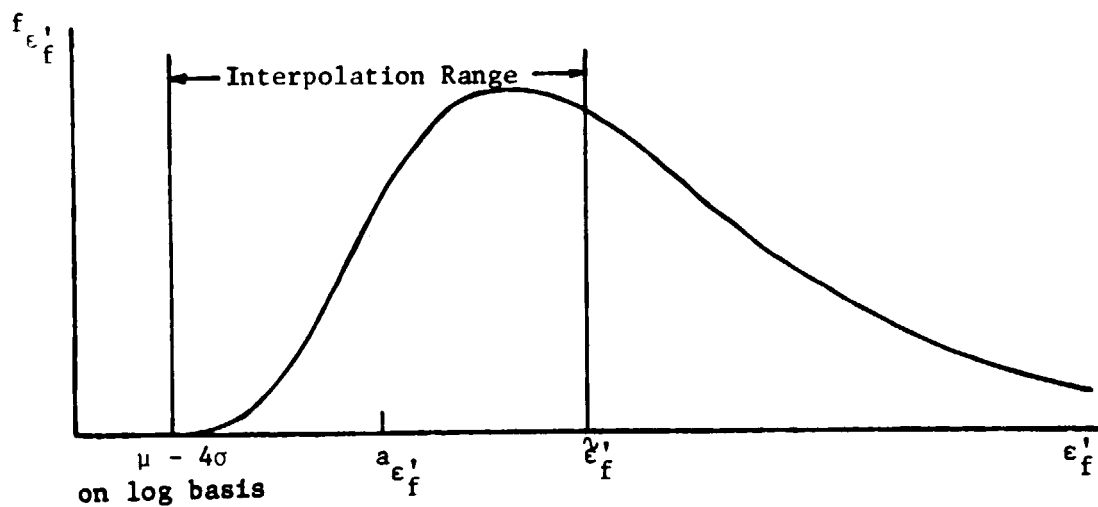
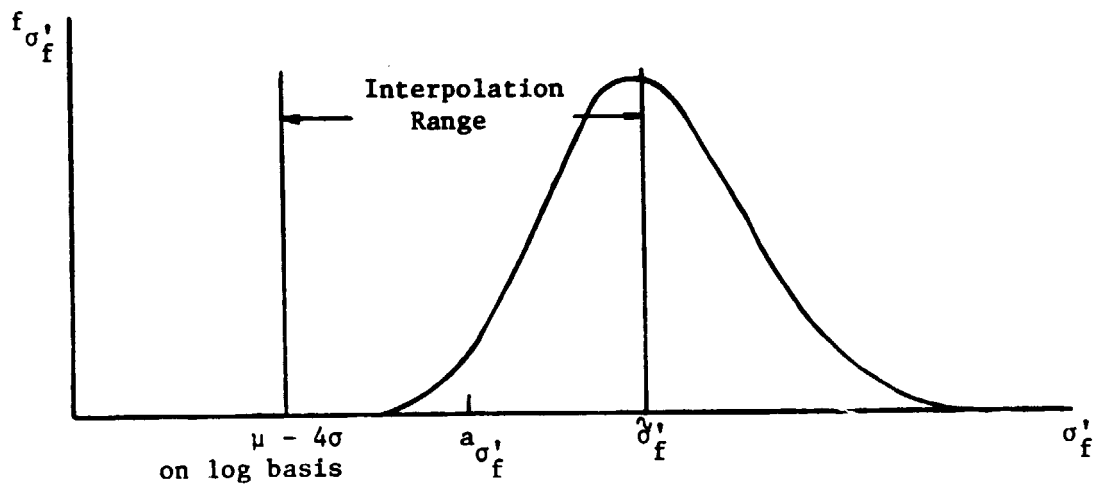
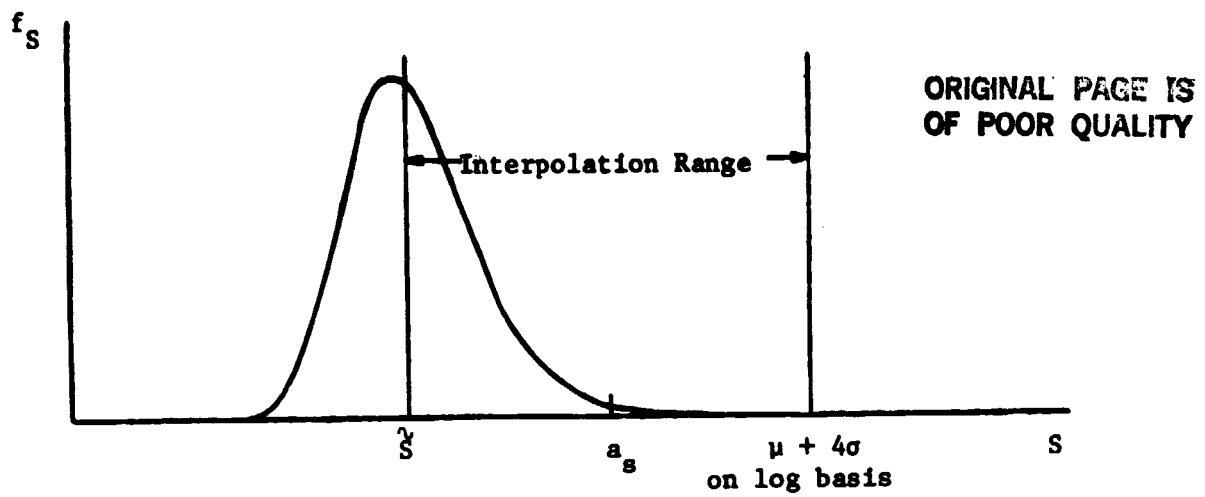


FIGURE 7. How the Expanding Point a and the Data Points were Chosen for Generating the Quadratic Polynomial Model.

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TABLE 1. Data Information for Example

(a) Random Variables

Parameter	Distribution Type	Median	Coefficient of Variance
S	lognormal	310.2 MPa ^a (45 ksi)	0.20
σ'_f	lognormal	1841 MPa (267 ksi)	0.05
ϵ'_f	lognormal	3.47	0.43
^a \bar{S} $\bar{S} = 414 \text{ MPa (60 ksi) for Case B, } \bar{S} = 551 \text{ MPa (80 ksi) for Case C}$			

(b) Constants

Parameter	Value	Parameter	Value
K_t	3.0	s	0.108
b	-0.0843	c	-0.9126
E	206820 MPa (30,000 ksi)	A	1518 MPa (220.4 ksi)
Note: $\sigma_o = 0$			

TABLE 2. Data Point Sets for Curve Fitting

Set Number	S, in MPa	σ'_f , in MPa	ϵ'_f	$\log_{10} N$, in cycles ^(b)
1 ^a	497.7	1674	2.069	3.859
2	310.2	1674	2.069	6.191
3	685.1	1674	2.069	2.935
4	497.7	1507	2.069	3.611
5	497.7	1841	2.069	4.149
6	497.7	1674	0.668	3.644
7	497.7	1674	3.470	3.984
Interpolation Range	310.2-685.1	1507-1841	0.668-3.47	2.935-6.191
^a Expanding point				
^b Computed using a local strain analysis program originally written by W. R. Brose.				

Now examine the quality of the polynomial expression. Consider the effective range for each independent variable. Using Eqs. 29 to 31, the values at $\partial S / \partial \sigma'_f = 0$, etc., are

$$(\sigma'_f, \epsilon'_f, \log N)^* = (-3290, 4.4, 6.1)$$

The value of $(\sigma'_f)^*$ is negative implying the nonlinear effect of σ'_f is very small. The value of $(\epsilon'_f)^*$ when compared with the interpolation range of ϵ'_f shown in Table 2 also suggests that the nonlinear effect of ϵ'_f is not significant. Thus, the quadratic model seems to be satisfactory for σ'_f and ϵ'_f in the chosen ranges of interpolation. The value of $(\log N)^*$ is close to the interpolation range implying a moderate nonlinear effect of $\log N$ in its chosen range.

The R-F algorithm was used to compute p_f for several values of N_0 . The results are shown in Table 3. It is noted that the highest value of $\log N_0$ is chosen to be 5.5, a value less than $(\log N)^*$, 6.1. This point is inside the interpolation range and reasonable p_f estimates are expected. There is no limitation for choosing lower $\log N_0$. The values listed in Table 3 are in the range where comparisons can be made with Monte Carlo results, i.e., where the lowest possible estimate of p_f is about 1%. Observing that all the estimated design point values in Table 3 are well within the interpolation range shown in Table 2 one notes that perhaps accuracy could be further improved by reducing the interpolation ranges.

Monte Carlo was used as an independent check on the quality of results of the proposed method. Unfortunately, each trial requires running the local strain analysis program with a random sample of S , σ'_f , and ϵ'_f . This process is extremely costly and inefficient. Therefore, only a sample of size 100 was obtained.

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TABLE 3. Results of the Reliability Analysis for Case A Using the
Proposed Method

		Design Point			Probability of failure, %
N_o (cycles)	$\log N_o$	S , in MPa	σ'_f , in MPa	ϵ'_f	
3,160	3.5	605	1779	2.88	0.026
10,000	4.0	520	1785	2.95	0.34
31,600	4.5	454	1793	3.04	2.2
100,000	5.0	407	1802	3.13	7.3
316,000	5.5	378	1810	3.21	14.2

Computed values of p_f from the R-F algorithm are plotted in Fig. 8. The empirical CDF generated from Monte Carlo method is also plotted on the same lognormal probability paper. Assuming that the p_f of the present method are accurate, the upper and lower 1% points for the maximum likelihood estimator (MLE), \hat{p} , of p are given. The calculation is similar to that of a confidence interval for the binomial. If the circled points in Fig. 8 are accurate, then there is a 98% chance that \hat{p} obtained from an experiment would lie in the interval shown. Even though the algorithm used to estimate p_f was really $(1 - \frac{1}{2})/n$, not exactly the MLE, the evidence of Fig. 8 suggests good agreement between the present method and Monte Carlo.

The median of S in this case was sufficiently small so that generally the deformation of the material is in the elastic range. However, because S has a large coefficient of variance, some plasticity is expected for samples having a higher than average stress. The plasticity effect (ϵ'_f term) could be important in highly stress specimens. Hence, the lower tail of the CDF of N is not a straight line as it would be on lognormal paper if S were deterministic. This exercise demonstrates that extrapolating p_f using Monte Carlo results will likely produce inaccurate estimates of p_f . This is a principal reason why the number of simulations should be larger when the p_f to be estimated is smaller and why Monte Carlo can be so expensive.

Case B

The data is the same as in Case A (See Table 1) but the median of nominal stress is increased to 414 MPa (60 ksi) so that more cyclic plasticity is expected at the notch.

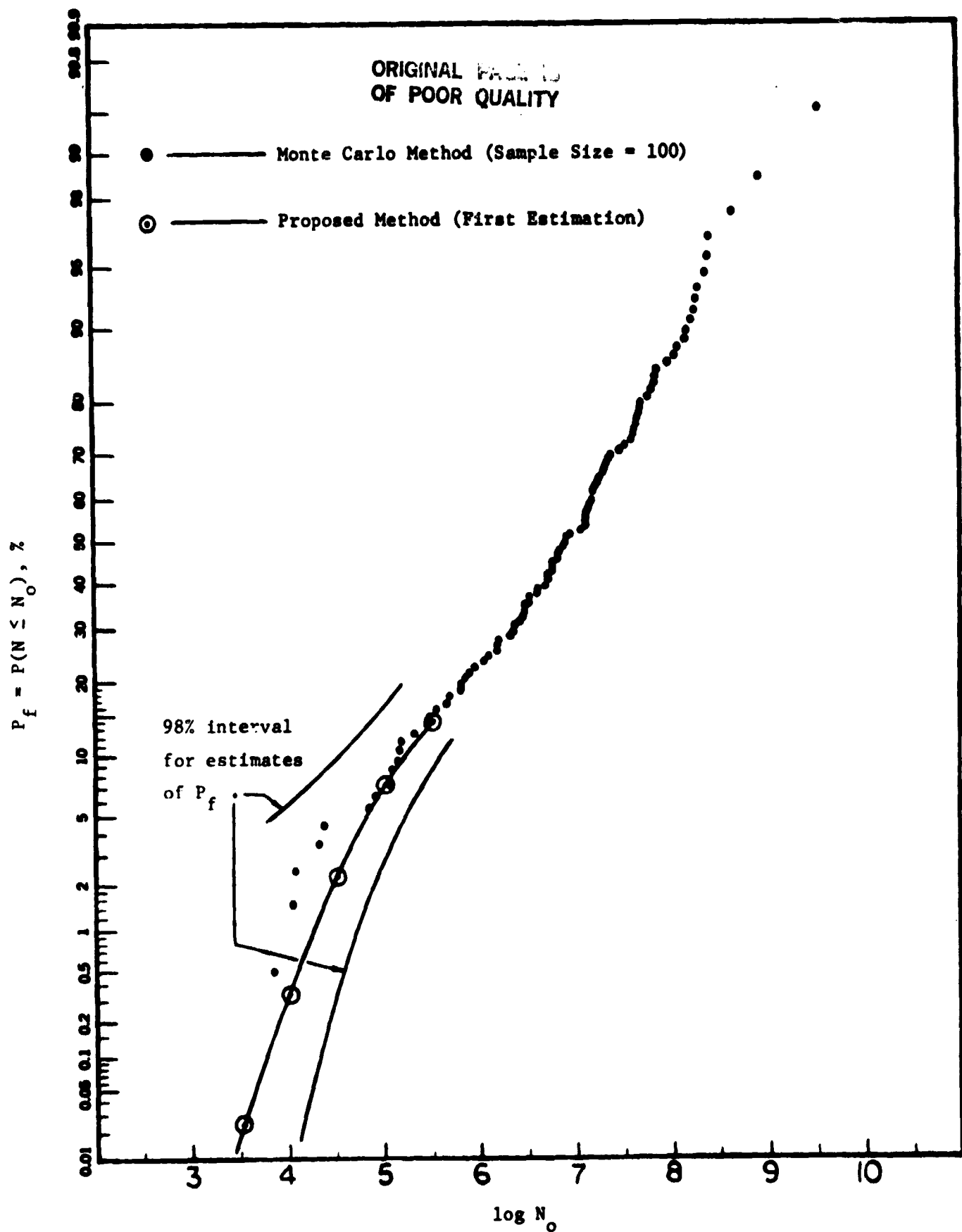


FIGURE 8. Reliability Analysis of Fatigue Life (Case A); A Comparison of the Present Method with Monte Carlo.

The data points for S are (414, 663, 913) MPa. The method for computing these was described in Fig. 7. Following the same procedures as in Case A, seven data points were obtained. The expanding point of log N is 3.0, and the data of log N ranges from 2.5 to 4.7.

The result of the curve fit, choosing S as the dependent variable, is

$$S = 663 + 0.217 (\sigma'_f - a_1) + 106 (\epsilon'_f - a_2) - 425 (\log N - a_3) + 0.000074 (\sigma'_f - a_1)^2 - 34.1 (\epsilon'_f - a_2)^2 + 164 (\log N - a_3)^2 \quad (38)$$

The values at $\partial S / \partial \sigma'_f = 0$, etc., are

$$(\sigma'_f, \epsilon'_f, \log N)^* = (207, 3.6, 4.3)$$

Again, both $(\sigma'_f)^*$ and $(\epsilon'_f)^*$ values are well-outside the interpolation range shown in Table 2 implying satisfactory effective ranges for σ'_f and ϵ'_f . However, $(\log N)^*$ is inside the interpolation range (2.5, 4.7), suggesting the possibility of a poor fit in some regions because of significant nonlinearity between S and log N in the interpolation region. Therefore, in forming the limit states of Eq. 38, values of log N₀ should be limited to, say 4.0 as higher errors may result for larger log N₀.

The design points and p_f for several values of log N₀ were computed using the same method as described for Case A. The results of this first approximation is shown in Table 4(a). Values of the p_f are plotted on Fig. 10 as the dashed line.

The fact that the $(\log N)^*$ value lies inside the interpolation range implies that the degree of nonlinearity between S and log N is significant in this range. A higher degree polynomial equation is required to improve the fit unless the range can be reduced. Suppose, for instance, the goal of the reliability analysis of log N₀ ranges from 3.0 to 4.5.

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The design point information generated from the first approximation (Table 4(a)) reveals that the interpolation ranges may be reduced to generate improved, second approximation. It should be emphasized that when only point estimate is required (e.g., $\log N_0 = 3.0$), the interpolation range for the second approximation can be made very small so that the quadratic model becomes a good approximation. In this study, however, the narrower interpolation range is chosen to be wide enough to cover the region of $\log N_0$ of interest in this study.

Using the design point information in Table 4(a), the new and reduced interpolation ranges are determined as shown in Table 4(b). The improved quadratic model becomes:

$$S = 551 + 0.212 (\sigma'_f - a_1) + 19.3 (\epsilon'_f - a_2) - 165 (\log N - a_3) + 0.000074 (\sigma'_f - a_1)^2 - 4.08 (\epsilon'_f - a_2)^2 + 55.1 (\log N - a_3)^2 \quad (39)$$

where the expanding point values can be found in Table 4(b). The values at $\partial S / \partial \sigma'_f = 0$, etc., are

$$(\sigma'_f, \epsilon'_f, \log N)^* = (390, 4.6, 5.2)$$

By comparing these values with the interpolation ranges shown in Table 4(b) it is found that the effective ranges for all three variables are satisfactory, suggesting that the model has been improved. To demonstrate the improvement made in the model, the exact and the approximate $S - N$ curves are drawn in Fig. 9 for both the first and the second approximation (Note: Values of σ'_f and ϵ'_f are at the expanding point). The figure clearly shows that the model is much better when $(\log N)^*$ lies outside the interpolation range suggesting that $(\log N)^*$, etc., which defines the effective range, is a convenient index of "measuring" the quality of the

TABLE 4. Results of the Reliability Analysis Using the First Approximating Model, and the Data Range for Generating Second Approximating Model (Case B).

(a) Results Using First Approximating Model

		Design Point			Probability of failure, %
N_o (cycles)	$\log N_o$	S , in MPa	σ'_f , in MPa	ϵ'_f	
1,000	3.0	658	1806	1.75	0.19
1,778	3.25	587	1811	1.95	1.14
3,162	3.5	539	1816	2.20	3.97
5,623	3.75	508	1820	2.47	8.93
10,000	4.0	490	1823	2.69	14.2

(b) Data Range for Generating Second Approximating Model

Variables	Data Range	Expanding Point
S , in MPa	655 - 448	551
σ'_f , in MPa	1806 - 1834	1820
ϵ'_f	1.75 - 2.80	2.275
$^a \log N$	3.15 - 4.58	3.686
^a Corresponding Values		

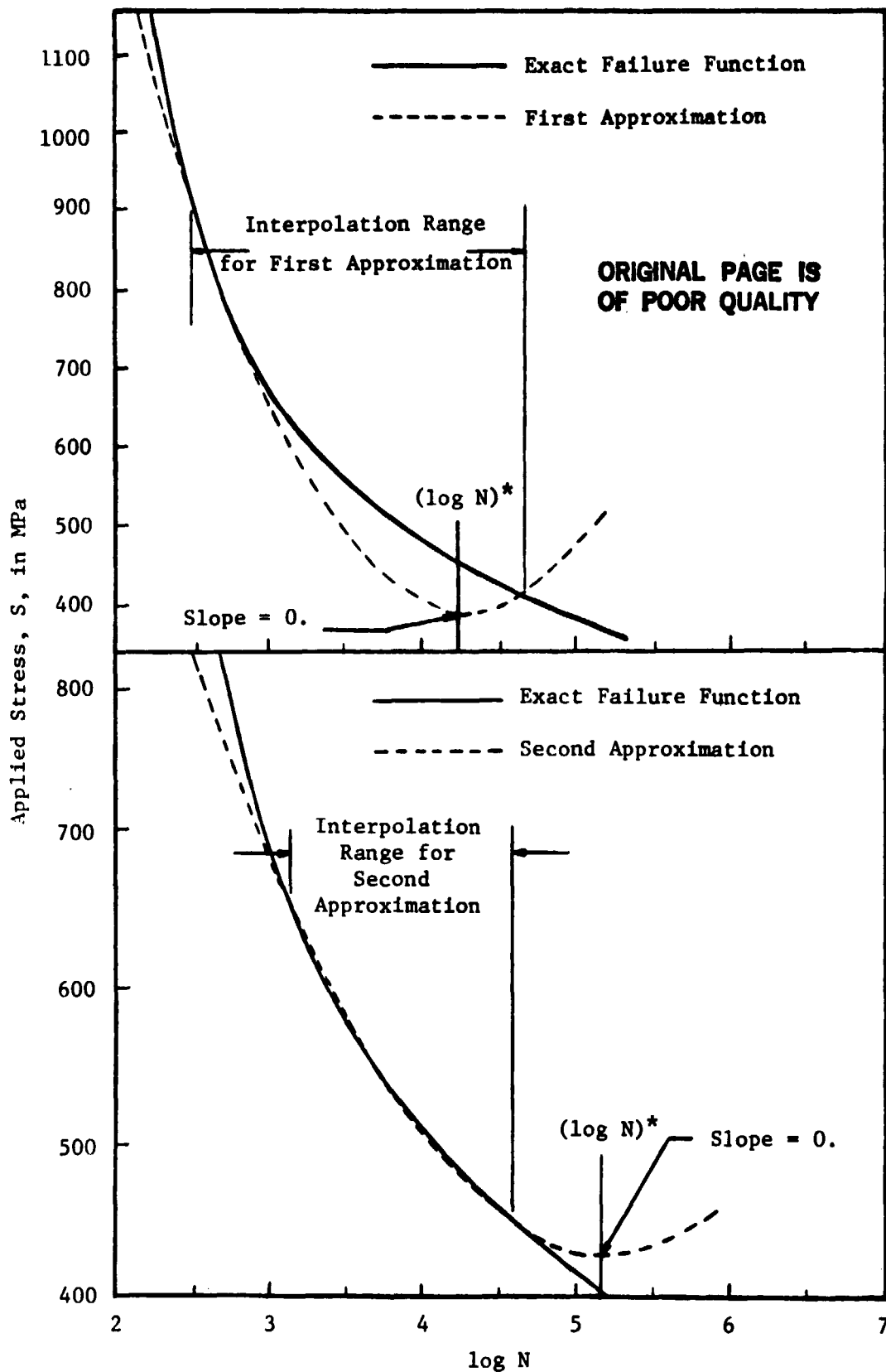


FIGURE 9. Comparison of the Exact and the Approximating Failure Function.

model, and the degree of nonlinearity in the failure function.

The new design points and the associated p_f using Eq. 39 are shown in Table 5. The design point values of ϵ'_f as well as the value of S when $\log N_0 = 3.0$ are outside the data ranges indicating the extrapolating values of Eq. 39 are being used. However, these values are well within the effective ranges and closer to the interpolation range. Therefore, it is not unreasonable to assume that good approximation can still be obtained.

Values of p_f of the improved model are also plotted on Fig. 10. Following the same procedure as in Case A, a 98% interval on the estimates of p_f is constructed which covers the Monte Carlo results. The improvement of the model seems justified. By using more appropriate data ranges or splitting the data range into two parts, accurate Rackwitz-Fiessler values of β can be obtained for a wide range of fatigue life. For a specified single value of design life, it is believed that no more than two tries are necessary to obtain satisfactory results.

Case C

For the previous examples, the median nominal stress is further increased to 551 MPa (80 ksi) so that even more cyclic plasticity is expected. Otherwise, the parameters are the same as in Cases A and B.

Applying the same methods as described in previous cases, the data points for S are (551, 885, 1218) MPa. The corresponding expanding point value of $\log N$ is 2.55, and the interpolation range of $\log N$ is from 2.05 to 3.48. The result of the curve fit is:

TABLE 5. Improved Results of the Reliability Analysis for Case B Using
the Proposed Method.

		Design Point			Improved p_f , %	First Estimates of p_f , %
N_o (cycles)	$\log N_o$	S, in MPa	σ'_f , in MPa	ϵ'_f		
1,000	3.0	700	1808	3.05	0.34	0.19
1,778	3.25	645	1810	3.08	1.12	1.14
3,162	3.5	596	1814	3.13	2.96	3.97
5,623	3.75	555	1817	3.18	6.43	8.93
10,000	4.0	520	1821	3.23	11.7	14.2
17,782	4.25	493	1825	3.28	18.2	-----
31,622	4.5	472	1828	3.32	24.7	-----

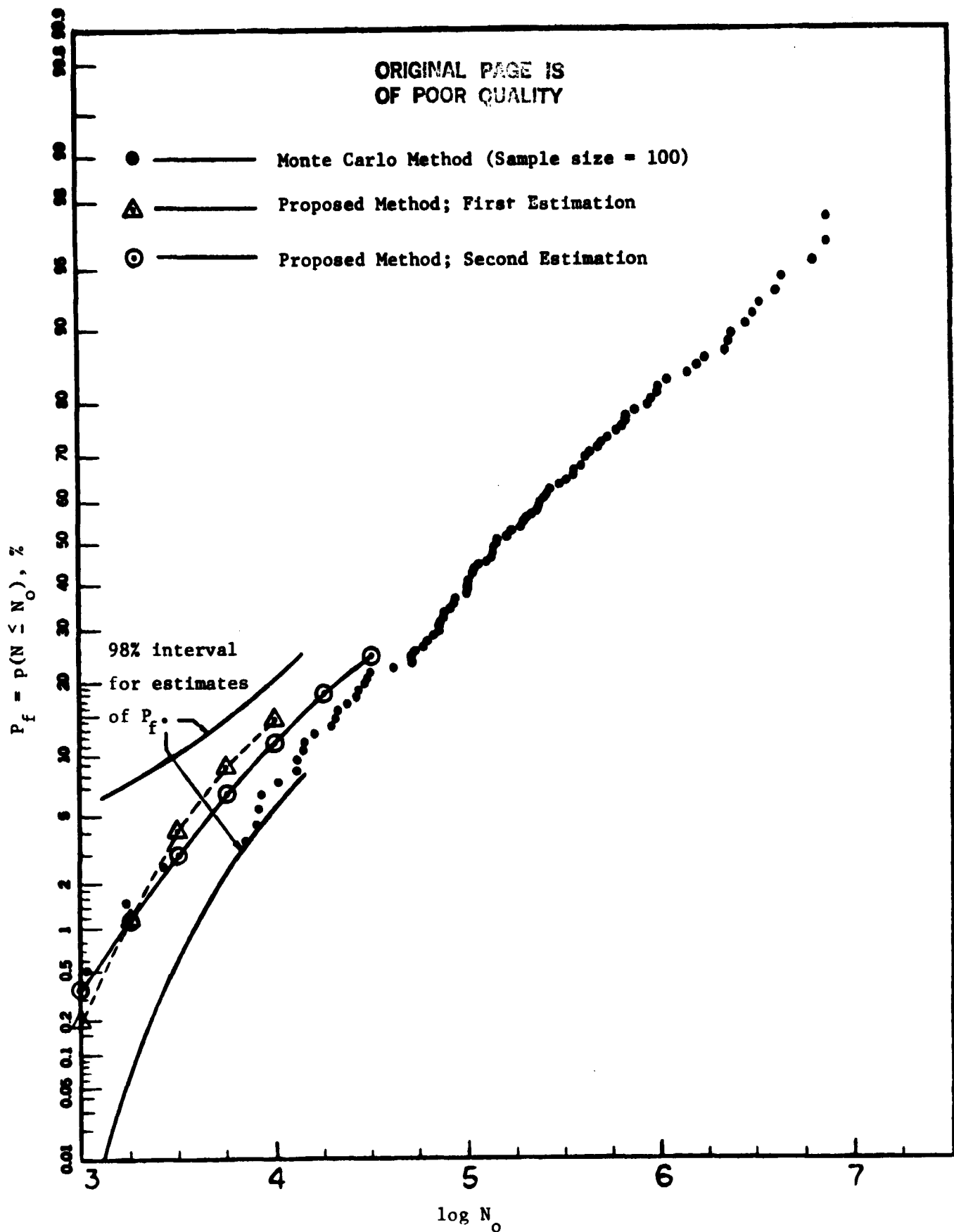


FIGURE 10. Reliability Analysis of Fatigue Life (Case B); A Comparison of the Present Method with Monte Carlo.

$$S = 885 + 0.133 (\sigma'_f - a_1) + 210 (\epsilon'_f - a_2) + 702 (\log N - a_3) \\ + 0.000012 (\sigma'_f - a_1)^2 - 77.1 (\epsilon'_f - a_2) + 370 (\log N - a_3)^2 \quad (40)$$

The values at $\partial S / \partial \sigma'_f = 0$, etc., are

$$(\sigma'_f, \epsilon'_f, \log N)^* = (-3870, 3.43, 3.5)$$

Thus, satisfactory effective ranges for σ'_f and ϵ'_f result. The value of $(\log N)^*$ is approximately equal to the value at right end of the interpolation range suggesting that the range of interpolation of S is too wide. To improve the model, a decision can be made to reduce the interpolation range either before or after the design points are estimated using Eq. 40. The method of improving the accuracy through range reduction process has been demonstrated in Case B, therefore it is not repeated here.

To obtain the first estimation of reliability analysis of N , several values of $\log N_0$ are chosen to form the limit states. The highest value is restricted to 3.25 which is somewhat less than the value of $(\log N)^*$. The design points and p_f using R-F algorithm are shown in Table 6. The values of ϵ'_f and σ'_f indicate that cyclic plasticity indeed plays the dominant role in the region of high p_f . p_f values are plotted in Fig. 11. A 98% interval on the estimates of p_f , and the Monte Carlo results are also plotted. It is noted that for this high stress case, lower tail CDF of N_0 should approach a straight line on the lognormal paper if the elasticity effect is being neglected. Thus, the result of the first estimation is reasonable.

SUMMARY

A numerical procedure is proposed for estimating risk for the case when the failure function is not available as a closed algebraic form. The safety index β is computed by the R-F algorithm after the

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TABLE 6. Results of the Reliability Analysis for Case C Using the Proposed Method.

		Design Point			Probability of failure, %
N_o (cycles)	$\log N_o$	S , in MPa	σ'_f , in MPa	ϵ'_f	
178	2.25	927	1825	1.25	0.015
316	2.5	786	1828	1.46	0.28
562	2.75	706	1831	1.75	1.8
1000	3.0	663	1833	2.06	5.7
1778	3.25	641	1834	2.30	10.5

failure function is approximated as a simple polynomial equation. The proposed method is described by the following step by step procedures (refer to Fig. 1):

1. Specify the random design factors.
2. Make a somewhat conservative but reasonable first guess of the design point to define the range of interpolation. Select several points from the range for each variable where a solution is required.
3. Solve the failure function, using computer analysis, to obtain a solution at each one of the point sets selected. The data sets for curve fitting are thereby provided.
4. Obtain a polynomial equation using a curve fitting technique. It is recommended that a quadratic equation without the interaction terms may be adopted for an efficient estimation.
5. Check the quality of the polynomial fit (e.g., define effective ranges) as an aid in forming the limit state where a reasonable agreement between the exact and the approximating equation can be anticipated. A reduction of the ranges of the selected points may be required if the failure function is highly nonlinear in the selected interpolation range.
6. Apply R-F algorithm to the approximating limit state to compute the design point and β .
7. Compare the design point values with the ranges of the selected points. To improve the approximation in the region of design point, go to Step 2, but adjust the values and ranges of the points for each variable. In general, accuracy can be improved by reducing the ranges which should still embrace the design point.
8. Estimate the probability of failure using $p_f = \Phi(-\beta)$.

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